Modeling the Electric Vehicle

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To determine how a commute would look if an electric car was driven instead of a gasoline car, a mathematical model for the car's motion is necessary. This allows us to take altitude and position data and translate it into power, allowing us to calculate the theoretical electric car's demands for the trip.

The model we use takes into account several forces and assumptions:

- $F_{motor} = ma$, where m is the mass of the car (and cargo) and a is the acceleration experienced as a result of the force exerted by the motors.
- $F_g = mg\sin\theta$, where m is the mass of the car (and cargo), g is the acceleration due to gravity (9.81 m/s^2) and θ represents the angle with the horizontal created by the change in altitude.
- $F_{rr} = C_{rr}mg$, where C_{rr} is a constant of rolling resistance, m is the mass of the car and cargo, and g is the acceleration due to gravity.
- $F_{ar} = \frac{1}{2}\rho v^2 C_d A$
    - v is the velocity of the car
    - C_d is the coefficient of drag of the car
    - A is the frontal area of the car
    - ρ is the density of air, which is calculated by:

      $p = p_0 \cdot \left(1 - \frac{L \cdot h}{T_0}\right)^{\frac{g \cdot m}{R \cdot L}}$

      $\rho = \frac{p \cdot M}{R \cdot T}$

      where:

        - sea level standard atmospheric pressure p_0 = 101325 Pa
        - sea level standard temperature T_0 = 288.15 K
        - Earth-surface gravitational acceleration g = 9.81 m/s^2.
        - temperature lapse rate L = 0.0065 K/m
        - universal gas constant R = 8.31447 J/(mol·K)
        - molar mass of dry air M = 0.0289644 kg/mol

These forces are all calculated, then converted to power using the formula $P = F_{total} \cdot v$, where F_{total} is the sum of the forces and v is the velocity of the car.

We also keep track of changes in internal resistance as the temperature of the battery changes, in order to accurately account for heat waste. To find the heat loss, we use:

- $Q_{cooling} = h \cdot A \cdot (T - T_A)$, where Q is the heat flow out of the car due to air cooling, $h = 10.45 - v - 10\sqrt{v}$, A is the exposed surface area of the battery, T is the current battery temperature and T_A is the air temperature.
- $P = I^2 \cdot R$, where P is the power lost to heat, I is the current and R is the resistance from the previous timestep.

- $Q_{tot} = m \cdot c \cdot \Delta T$, where Q_{tot} is the net increase in heat energy in the battery over the timestep, m is the mass of the battery, c is the specific heat of the battery and ΔT is the change in temperature.
- $R(T) = T^{-0.02573644} - 0.8555693034$, where R(T) is the internal resistance and T is the temperature. This equation is fit to a curve in [cite textbook].

The model assumes:

- There is a -0.35 kW constant discharge due to car accessories.
- If there is regenerative braking (covered in F_{motor}), the car takes in a constant proportion of .35 of the possible recoverable energy. All braking is considered regenerative.
- We assume a constant discharge efficiency of .80 due to drivetrain inefficiencies.
- If the car is stopped, it uses only the constant discharge.
- Since the GPS data is not continuous and has discrete timesteps, we assume that the power output is constant between steps.

When the model is implemented, it compares somewhat closely to measured, actual data. There are several sources of error in the assumptions and in satellite error, accounting for the imperfections of the model. Here is the model compared against a real trip with power measurements: